

# Simple model for QCD analysis of the proton helicity structure

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In this paper we use the experimental data to obtain the polarized parton distribution functions (PPDFs) in the LO and NLO approximations. The analysis is based on the Jacobi polynomials expansion of the polarized structure function (PSF). Our calculations for polarized parton distribution functions based on the Jacobi polynomials method are in good agreement with the other theoretical models.

## I. INTRODUCTION

The theoretical and experimental status on the spin structure of the nucleon has been discussed in great detail in several recent reviews (see, e.g., Refs. [1, 2, 3, 4]. Deeply inelastic scattering provides a clean way to extract the parton densities of nucleons. After the initial observation that the nucleon spin is not formed by the quarks dominantly [5], detailed measurements of the polarized structure functions followed during the last 20 years. The central question concerns now the parton distribution functions and their scale evolution rather than just their first moment. Since the nucleon spin receives also contributions from the angular momentum of the quarks and gluons, these degrees of freedom have also to be studied. During the recent years several comprehensive analysis of the polarized deep inelastic scattering (DIS) data, based on next-to-leading-order quantum chromodynamics have performed. In these analysis the polarized parton density functions are either written in terms of the well-known parameterizations of the unpolarized PDFs or parameterized independently, and the unknown parameters are determined by fitting the polarized DIS data.

Determination of parton distributions in a nucleon in the framework of quantum chromodynamics (QCD) always involves some model-dependent procedure. Instead of relying on mathematical simplicity as a guide, we take a viewpoint in which the physical picture of the nucleon structure is emphasized. That is, we consider the model for the nucleon which is compatible with the description of the bound state problem in terms of three constituent quarks. We adopt the view that these constituent quarks in the scattering problems should be regarded as the valence quark clusters rather than point-like objects. They have been referred to as *valons*. The idea of nucleon as a bound state of three quarks was presented for the first time in Ref. [6]. In the valon model, the proton consists of two “up” and one “down” valons. These valons thus, carry the quantum numbers of the respective valence quarks. Hwa and et al. [7] found evidence for the valons in the deep inelastic neutrino scattering data, suggested their existence and applied it to a variety of phenomena. In [8] unpolarized PDFs and hadronic

structure functions in the NLO approximation were extracted. In Ref. [9] the polarized valon model is applied to determine the quark helicity distributions and polarized proton structure functions in the NLO approximation by using the Bernstein polynomial approach. The extraction of the quark helicity distributions is one of the main tasks of the semi-inclusive deep inelastic scattering (SIDIS) experiments (HERMES [10], COMPASS [11], SMC[12]) with the polarized beam and target. Recently in Ref. [13, 14] the polarized valon model was applied and analyzed the flavor-broken light sea quark helicity distributions with the help of a Pauli-blocking ansatz. The reported results of this paper are in good agreement with the HERMES experimental data for the quark helicity distributions in the nucleon for up, down, and strange quarks from semi-inclusive deep-inelastic scattering [10].

Since very recently experimental data are available from the HERMES collaboration [15] for the spin structure function  $g_1$ , therefore there is enough motivation to study and utilize the spin structure and quark helicity distributions extracted via the phenomenological model. We performed the first recalculation for the spin structure function  $g_1$  up to NLO [16] using the Jacobi polynomials expansion[17]. This paper is based on the Ref.[16].

The plan of the paper is to give a brief review of the theoretical background of the QCD analysis in two loops. The method of the QCD analysis of polarized structure function, based on Jacobi polynomials are written down in Section 3. A description of the procedure of the QCD fit of  $g_1$  data and results are illustrated in Section 4.

## II. THE THEORETICAL BACKGROUND

Let us define the MELLIN moments for the polarized structure function  $g_1^p(x, Q^2)$ :

$$g_1^p(N, Q^2) = \int_0^1 x^{N-1} g_1^p(x, Q^2) dx . \quad (1)$$

The contribution to the structure function  $g_1(N, Q^2)$  up to NLO can be represented in terms of the po-

larized parton densities and the coefficient functions  $\Delta C_i^N$  in the MELLIN -N space by [2]

$$\begin{aligned} g_1^p(N, Q^2) = & \frac{1}{2} \sum_q e_q^2 \left\{ \left( 1 + \frac{\alpha_s}{2\pi} \Delta C_q^N \right) [\Delta q(N, Q^2) + \right. \\ & \left. \Delta \bar{q}(N, Q^2)] + \frac{\alpha_s}{2\pi} 2 \Delta C_g^N \Delta g(N, Q^2) \right\}, \end{aligned} \quad (2)$$

in this equation the NLO running coupling constant is given by

$$A_s = \frac{1}{\beta_0 \ln Q^2 / \Lambda_{MS}^2} - \frac{\beta_1 \ln(\ln Q^2 / \Lambda_{MS}^2)}{\beta_0^3 (\ln Q^2 / \Lambda_{MS}^2)^2}. \quad (3)$$

The symbol  $A_s$  denotes the strong coupling constant normalized to  $A_s = \alpha_s / (4\pi)$ . Notice that in the above the numerical expressions for  $\beta_0, \beta_1$  are

$$\begin{aligned} \beta_0 &= 11 - 0.6667f, \\ \beta_1 &= 102 - 12.6667f, \end{aligned} \quad (4)$$

where  $f$  denotes the number of active flavors. In our calculation, we choose  $Q_0 = 1 \text{ GeV}^2$  as a fixed parameter and  $\Lambda$  is an unknown parameter which can be obtained by fitting to experimental data.

In Eq. (2),  $\Delta q(N, Q^2) = \Delta q_v(N, Q^2) + \Delta \bar{q}(N, Q^2)$ ,  $\Delta \bar{q}(N, Q^2)$  and  $\Delta g(N, Q^2)$  are moments of the polarized parton distributions in a proton.  $\Delta C_q^N$ ,  $\Delta C_g^N$  are also the  $N$ -th moments of spin-dependent Wilson coefficients given in Ref.[16].

According to improved polarized valon model framework, determination of the moments of parton distributions in a proton can be done strictly through the moments of the polarized valon distributions.

The moments of PPDFs are denoted respectively by:  $\Delta u_v(N, Q^2)$ ,  $\Delta d_v(N, Q^2)$ ,  $\Delta \Sigma(N, Q^2)$  and  $\Delta g(N, Q^2)$ . Therefore, the moments of the polarized  $u, d, \Sigma$  and  $g$  density functions in a proton are:

$$\Delta u_v(N, Q^2) = 2 \Delta M^{NS}(N, Q^2) \times \Delta M'_{U/p}(N), \quad (5)$$

$$\Delta d_v(N, Q^2) = \Delta M^{NS}(N, Q^2) \times \Delta M'_{D/p}(N), \quad (6)$$

$$\Delta \Sigma(N, Q^2) = \Delta M^S(N, Q^2) (2 \Delta M''_{U/p}(N) + \Delta M''_{D/p}(N)), \quad (7)$$

$$\Delta g(N, Q^2) = \Delta M^{gq}(N, Q^2) (2 \Delta M'_{U/p}(N) + \Delta M'_{D/p}(N)). \quad (8)$$

In the above equations  $M'_{j/p}(N)$  and  $M''_{j/p}(N)$  are the moments of polarized valon distributions, which introduced in Ref.[16]. It is obvious that the final

form for  $g_1(N, Q^2)$  involves some unknown parameters. If the parameters can be obtained then the computation of all moments of the PPDFs and the polarized structure function,  $g_1(N, Q^2)$ , are possible.

### III. THE METHOD OF THE QCD ANALYSIS

One of the simplest and fastest possibilities in the PSF reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited for this purpose since they allow one to factor out an essential part of the  $x$ -dependence of the structure function into the weight function [18]. Thus, given the Jacobi moments  $a_n(Q^2)$ , a structure function  $f(x, Q^2)$  may be reconstructed in a form of the series [19]-[23]

$$xf(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} a_n(Q^2) \Theta_n^{\alpha, \beta}(x), \quad (9)$$

where  $N_{max}$  is the number of polynomials and  $\Theta_n^{\alpha, \beta}(x)$  are the Jacobi polynomials of order  $n$ ,

$$\Theta_n^{\alpha, \beta}(x) = \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) x^j, \quad (10)$$

where  $c_j^{(n)}(\alpha, \beta)$  are the coefficients that expressed through  $\Gamma$ -functions and satisfy the orthogonality relation with the weight  $x^\beta (1-x)^\alpha$  as following

$$\int_0^1 dx x^\beta (1-x)^\alpha \Theta_k^{\alpha, \beta}(x) \Theta_l^{\alpha, \beta}(x) = \delta_{k,l}, \quad (11)$$

For the moments, we note that the  $Q^2$  dependence is entirely contained in the Jacobi moments

$$\begin{aligned} a_n(Q^2) &= \int_0^1 dx xf(x, Q^2) \Theta_n^{\alpha, \beta}(x) \\ &= \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) f(j+2, Q^2), \end{aligned} \quad (12)$$

obtained by inverting Eq.( 9), using Eqs. (10, 11) and also definition of moments,  $f(j, Q^2) = \int_0^1 dx x^{j-2} xf(x, Q^2)$ .

Using Eqs. (9-12) now, one can relate the PSF with its Mellin moments

$$\begin{aligned} x g_1^{N_{max}}(x, Q^2) &= x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} \Theta_n^{\alpha, \beta}(x) \times \\ &\quad \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) g_1(j+2, Q^2), \end{aligned} \quad (13)$$

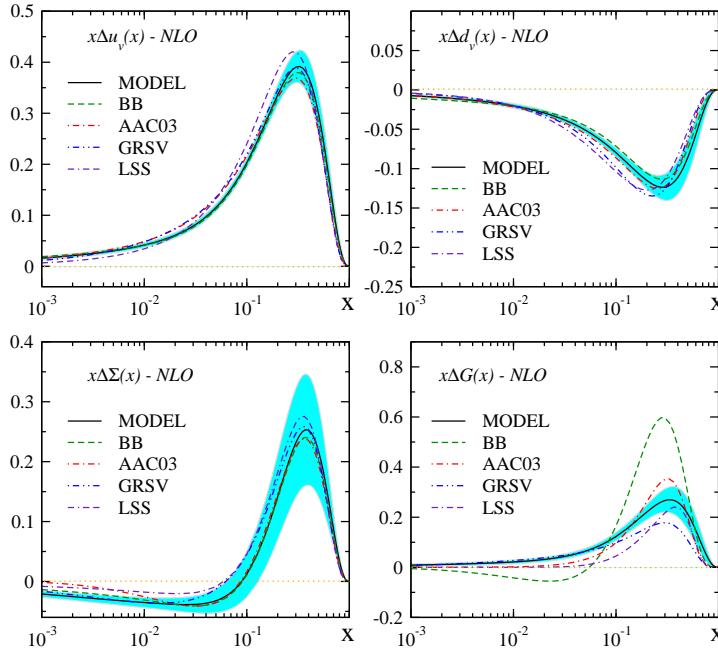


FIG. 1: NLO polarized parton distributions at the input scale  $Q_0^2=1.0$  GeV $^2$  compared to results obtained by BB model (dashed line) (ISET=3)[38], AAC (dashed-dotted line) (ISET=3)[39], GRSV (dashed-dotted dotted line) (ISET=1)[40] and LSS (dashed-dashed dotted line) (ISET=1)[41].

where  $g_1(j+2, Q^2)$  are the moments of polarized structure function.  $N_{max}$ ,  $\alpha$  and  $\beta$  have to be chosen so as to achieve the fastest convergence of the series on the R.H.S. of Eq. (13) and to reconstruct  $xg_1$  with the required accuracy. In our analysis we use  $N_{max} = 9$ ,  $\alpha = 3.0$  and  $\beta = 0.5$ . The same method has been applied to calculate the nonsinglet structure function  $xF_3$  from their moments [24-31].

Obviously the  $Q^2$ -dependence of the polarized structure function is defined by the  $Q^2$ -dependence of the moments.

#### IV. THE PROCEDURE OF THE QCD FIT AND RESULTS

For the QCD analysis presented in Ref. [16] the following data sets were used: the HERMES proton data [15, 32], the SMC proton data [33], the E143 proton data [34], the EMC proton data [35, 36]. In the fitting procedure, using the CERN subroutine MINUIT [37], we defined a global  $\chi^2$  for all the experimental

data points and found an acceptable fit with minimum  $\chi^2/\text{d.o.f.} = 0.978$  in the LO case and  $\chi^2/\text{d.o.f.} = 0.933$  in the NLO case. In Table. 2 of Ref.[16] we presented and compared the results which are based on Bernstein and Jacobi approaches.

In Figures 1 the parton distribution functions in next-to-leading order for all sets of parameterizations [38, 39, 40, 41] and their errors are presented at the starting scale  $Q_0^2$ .

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